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The spin and pseudospin symmetries in the relativistic formalism: Similarities and differences

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Abstract. The effects of the symmetry breaking terms, spin-orbit potential (SOP) and pseudospin-orbit potential (PSOP), in the spin and pseudospin symmetries, respectively, are studied in a comparative way. The analytical properties of the small component F of the Dirac spinor for pseudospin doublets (PSDs) are investigated around the singularity point of the PSOP. We show that the PSOP and the pseudocentrifugal barrier must be appropriately related to each other to describe adequately the wave functions in the nuclear surface, whereas it is not the case for the centrifugal barrier and the SOP. We also determine a modified PSOP smaller than the PSOP when the parameter $\bar{\kappa}$, appearing in it, is left to vary in the domain of real numbers. The inclusion of this modified PSOP allows us to define a continuous way that connects the two states of a PSD as $\bar{\kappa}$ varies continuously between the physical (integer) values of $\bar{\kappa}$ corresponding to these states. Our results indicate that whereas the SOP can be considered as a small spin symmetry breaking term that allows a simple explanation of the spin symmetry, the consideration of the PSOP as the pseudospin symmetry breaking term presents serious difficulties. Thus, we propose a new strategy to explain, in a simple way, the quasi-degeneracy of the PSDs.

PACS. 24.10.Jv Relativistic models – 21.60.Cs Shell model – 21.10.Pc Single-particle levels and strength functions – 24.80.+y Nuclear tests of fundamental interactions and symmetries

1 Introduction

1.1 The spin and pseudospin symmetries

The concept of pseudospin symmetry (PSS) appeared in nuclear physics more than 30 years ago to rename the single-particle (SP) levels in the shell model [1,2], but only recently, this symmetry has been considered as a relativistic symmetry [3–10], claiming that it is not possible to explain it properly in the non-relativistic framework. Thus, in the recent years, many other works have studied also the PSS using the relativistic framework through a Dirac equation with appropriate potentials fitted to reproduce certain properties of finite nuclei [10,11] or through the relativistic Hartree [12–16] or Hartree-Fock [17,18] approximations.

The PSS was originally associated to the frequently observed quasi-degeneracy of pseudospin doublets in both spherical and deformed nuclei. Two SP states labelled by "a" and "b" make a pseudospin doublet (PSD) if their radial, orbital, and total-angular-momentum quantum numbers are related by the equations $n_b = n_a - 1$, $l_b = l_a + 2$, and $j_b = j_a + 1 = l_a + 3/2$, respectively. In the pseudospin formalism, the same pseudo-orbital angular momentum $\tilde{l} = (2j - l)$ is assigned to both states of a PSD, and the total angular momentum of these states is given by $j = \tilde{l} \mp 1/2$. We shall say a PSD exhibits PSS if their two pseudospin (PS) partners have the same energy. In the same way, we shall say that there is spin symmetry (SS) if the two partners of a spin doublet (SD) have the same energy. In this work, we investigate different aspects of the spin and pseudospin symmetries in the Dirac-Hartree approximation.

1.2 The Dirac-Hartree Approximation (DHA)

In the DHA (where the tensor contribution of the vector mesons is neglected), the SP states are obtained from a

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Dirac equation that can be written as

$$[-i\boldsymbol{\alpha}\cdot\boldsymbol{\nabla}+\beta(M+\Sigma_S)+\Sigma_0]\psi(\vec{r})=E\psi(\vec{r}), \quad (1)$$

where, $E = M + \epsilon$ is the relativistic energy, M is the nucleon mass, Σ_S is the scalar self-energy coming from the scalar σ-meson and Σ_0 is the vector self-energy coming from the vector ω and ρ mesons and the Coulomb field. For spherical nuclei (to which we restrict ourselves in this work), the nucleon Dirac spinor $\psi(\vec{r})$ can be written, in standard notation, as

$$\psi(\vec{r})=\frac{1}{r}\begin{pmatrix} iG(r)y^m_{jl}(\theta,\phi)\\ F(r)y^m_{j\tilde{l}}(\theta,\phi)\end{pmatrix}, \quad (2)$$

where $\frac{G(r)}{r}$ and $\frac{F(r)}{r}$ represent the radial parts of its big and small components, respectively. Notice that $\tilde{l}=l\pm1$ appears in the small component of the spinor.

By substituting eq. (2) into eq. (1), one can get the following Dirac equation for the G and F components:

$$\begin{aligned}\frac{\mathrm{d}}{\mathrm{d}r}G(r)&=-\frac{\kappa}{r}\,G(r)+W\,F(r),\\ \frac{\mathrm{d}}{\mathrm{d}r}F(r)&=V\,G(r)+\frac{\kappa}{r}\,F(r).\end{aligned} \quad (3)$$

In this equation,

$$\kappa\equiv(2j+1)(l-j)=j(j+1)-\tilde{l}(\tilde{l}+1)+1/4, \quad (4)$$

whereas

$$V\equiv\Sigma_S+\Sigma_0-\epsilon,\qquad W\equiv2M+\Sigma_S-\Sigma_0+\epsilon \quad (5)$$

are energy-dependent potentials.

From the Dirac equation (3), one can get the two following equivalent second-order differential equations for the G and F components of the nucleon Dirac spinor:

$$-G''+\left[\frac{W'}{W}\left(\frac{G'}{G}+\frac{\kappa}{r}\right)+\frac{l(l+1)}{r^2}+VW\right]G=0, \quad (6)$$

$$-F''+\left[\frac{V'}{V}\left(\frac{F'}{F}-\frac{\kappa}{r}\right)+\frac{\tilde{l}(\tilde{l}+1)}{r^2}+VW\right]F=0, \quad (7)$$

where the following relations:

$$l(l+1)=\kappa(\kappa+1) \quad (8)$$

and

$$\tilde{l}(\tilde{l}+1)=\kappa(\kappa-1) \quad (9)$$

hold, in accordance with eq. (4), and the quantity

$$VW=2MV+2\epsilon\Sigma_0+(\Sigma_S^2-\Sigma_0^2)-\epsilon^2 \quad (10)$$

represents an effective state-dependent potential.

The quantum number κ takes different values for the two states of each spin and pseudospin doublets. Thus, in eqs. (6) and (7), the terms proportional to κ, to which we shall refer to as the spin-orbit potential (SOP) and pseudospin-orbit potential (denoted as PSOP or also as U_κ), respectively, or, simply, the $G-\kappa$ and $F-\kappa$ terms, explicitly break the degeneracy of the two partners of a SD or a PSD. In what follows, we shall designate by "a" and "b" the two states of a SD or PSD with $\kappa<0$ and $\kappa>0$, respectively. In refs. [14,15], it has been shown that whereas two states of a SD can be connected by a continuous way as κ (considered as a real number), in eq. (6), varies continuously from κ_a to κ_b, it is not possible for two states of a PSD as κ varies in eq. (7).

The PSS has been considered slightly broken in nuclei by some authors [6–9] because the $F-\kappa$ term in eq. (7) is small. More precisely, the condition required by these authors is that the magnitude of the $F-\kappa$ term should be much smaller than the pseudocentrifugal barrier (PCB), *i.e.*, $|U_\kappa\equiv\frac{V'}{V}\frac{\kappa}{r}|\ll U_{\tilde{l}}\equiv\frac{\tilde{l}(\tilde{l}+1)}{r^2}$. This condition can be considered less restrictive than the condition $\Sigma_S\simeq-\Sigma_0$ considered in refs. [3,5].

The aim of this work is: A) to study, in a comparative way, the effects of the symmetry breaking κ terms in the spin and pseudospin symmetries; B) to investigate the analytical properties of the small component F of the Dirac spinor for PSDs around the singularity point of the PSOP; C) to show that the PSOP and the PCB are both essential to describe adequately the nucleon wave functions in the nuclear surface where r_0, the singularity point of the PSOP, is localized; D) to determine the conditions that allow to find a continuous way to connect two states of a PSD as κ varies continuously, as a real number, from κ_a to κ_b; E) to show that the relations $\epsilon_a\simeq\epsilon_b$ and $F_a\simeq F_b$ for the two PS partners in finite nuclei are not directly dependent on each other; F) to investigate whether it is possible to find a term smaller than the $F-\kappa$ term responsible for the small splittings of the PSDs in nuclei; G) to present a new strategy to explain the PSS.

In sects. 2 and 3, we investigate the effects of the spin and pseudospin symmetry breaking terms (extending them to real values of κ) on the energies and the components G and F of the SP Dirac spinor corresponding to the spin and pseudospin doublets, analysing their differences in detail. In sect. 4, we identify a modified PSS breaking term that, in some cases, is weaker than the PSOP. In sect. 5, a new strategy to explain the PSS, valid in the relativistic as well as in the non-relativistic formalism, is proposed. In sect. 6 we discuss to which extent our results have a general character and can be applied to the spin and pseudospin doublets of nuclei different from the ^{40}Ca nucleus treated in the present paper. The conclusions are drawn in sect. 7.

2 Equation for the G and F components of the Dirac spinor for real values of κ

In the Dirac equation and, consequently, in eqs. (6) and (7), only integer values of κ, both negative (κ_a) and positive (κ_b), have physical meaning. However, to study

the $G-\kappa$ and the $F-\kappa$ terms, from a purely mathematical point of view, one may also consider equations with the same structure of eqs. (6) and (7) but allowing a continuous variation of the parameter κ (maintaining the physical values of l in eq. (6) and $\tilde{l}$ in eq. (7) corresponding to the physical integer values of κ according to eqs. (4), (8) and (9)). This real parameter will be designated as $\bar{\kappa}$ in what follows. For clarity, we write explicitly in terms of $\bar{\kappa}$ the two equations (6) and (7) for the G and F functions, respectively, as

$$-G'' + \left[\frac{W'}{W}\left(\frac{G'}{G} + \frac{\bar{\kappa}}{r}\right) + \frac{l(l+1)}{r^2} + VW\right] G = 0. \quad (11)$$

$$-F'' + \left[\frac{V'}{V}\left(\frac{F'}{F} - \frac{\bar{\kappa}}{r}\right) + \frac{\tilde{l}(\tilde{l}+1)}{r^2} + VW\right] F = 0, \quad (12)$$

where l and $\tilde{l}$ satisfy eqs. (8) and (9), respectively.

Notice that the character, integer or real, of κ in the spin and pseudospin symmetry breaking terms would not be essential if these terms were really small. If this were the case, eqs. (11) and (12) with $\bar{\kappa}$ real would continue having an important physical meaning. For example, the case corresponding to $\bar{\kappa} = 0$ would represent the situation of exact spin or pseudospin symmetry for which $\epsilon_a = \epsilon_b$ and $G_a \propto G_b$ or $F_a \propto F_b$, respectively. The case of exact symmetry, although it corresponds to a non-physical situation, would allow to understand, in a mathematical way, why the approximate physical symmetry does happen in finite nuclei. The conclusion obtained in refs. [6–9] that the PSS breaking term is small implicitly means that the character, integer or real, of κ is not essential. Reversing the previous arguments, the study of the solutions of eqs. (11) and (12) with $\bar{\kappa}$ real can shed light on the genuine relevance of the symmetry breaking terms to the realization of the spin and pseudospin symmetries, respectively. Equations (11) and (12) can be studied directly but, from the physical point of view, the corresponding equivalent Dirac equations are more relevant.

3 Equivalent Dirac equations

Each of eqs. (11) and (12) can be easily obtained from two different Dirac equations that we shall study in detail in what follows. As we shall see, if the $G-\kappa$ or $F-\kappa$ term were small, one of these equations would connect continuously, as $\bar{\kappa}$ varies, the component G or F of the two states of a SD or PSD, respectively, whereas the other, would connect both components. Let us study first the case corresponding to eq. (11) for the component G.

3.1 Dirac equations equivalent to eq. (11) for G and the spin symmetry

Choice 1. Equation (11) can be obtained from the Dirac equation (3) by replacing V by $\bar{V}$, defined by the equations

$$\bar{V}(r) = V(r) + \Delta V(r), \quad \Delta V(r) = -\frac{W'}{W^2}\frac{\kappa - \bar{\kappa}}{r}. \quad (13)$$

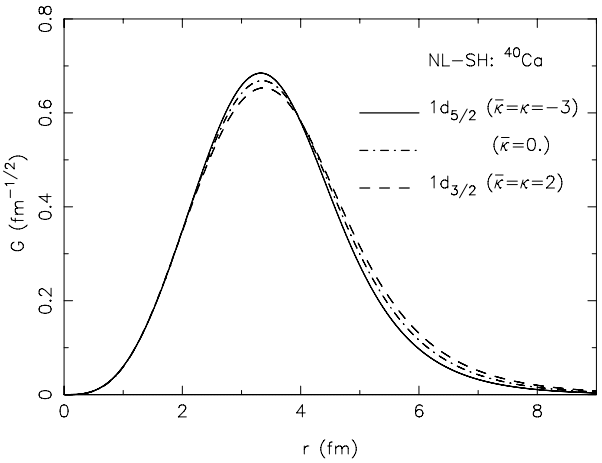

Fig. 1. The physical wave functions G_a and G_b, solutions of eq. (11) or eq. (14), with $\bar{\kappa} = \kappa_a$ (solid line) and $\bar{\kappa} = \kappa_b$ (dashed line), respectively, for the $1d$ neutron spin-orbit doublet of the ^{40}Ca nucleus and the NL-SH parameter set [20]. The dash-dotted curve represents the G function for the case $\kappa = -3$ and $\bar{\kappa} = 0$, *i.e.*, the non-physical function $G_a(\bar{\kappa} = 0.)$. Notice that a curve proportional to $G_a(\bar{\kappa} = 0.)$ would be obtained for $\kappa = 2$ and $\bar{\kappa} = 0$, corresponding to the non-physical function $G_b(\bar{\kappa} = 0.)$.

The Dirac equation including ΔV reads

$$\begin{aligned} \frac{\mathrm{d}}{\mathrm{d}r}G(r) &= -\frac{\kappa}{r}\,G(r) + W\,F(r), \\ \frac{\mathrm{d}}{\mathrm{d}r}F(r) &= (V + \Delta V)\,G(r) + \frac{\kappa}{r}\,F(r). \end{aligned} \quad (14)$$

Equation (14) allows us to write the component F in terms of the function G as

$$F = \frac{1}{W}\left(G' + \frac{\kappa}{r}G\right). \quad (15)$$

Equations (11) and (15) are equivalent to the Dirac equation (14) for any real value of $\bar{\kappa}$. For $\bar{\kappa} = \kappa$ (integer), eq. (14) and also eqs. (11) and (15) are equivalent to the Dirac equation (3). It is worth noting that for each value of G obtained from eq. (14) for a given value of $\bar{\kappa}$, we have two functions F, $F_a(\bar{\kappa})$ and $F_b(\bar{\kappa})$, corresponding to $\kappa = \kappa_a$ and $\kappa = \kappa_b$, respectively, with the same energy ($\epsilon_a(\bar{\kappa}) = \epsilon_b(\bar{\kappa})$).

As was shown in ref. [15], the G component and the energies of the two physical states a and b can be connected by a continuous way as $\bar{\kappa}$ varies continuously from $\bar{\kappa} = \kappa_a$ to $\bar{\kappa} = \kappa_b$. This means that the SP energy ϵ and the component G in eqs. (11) and (14) vary continuously as a function of $\bar{\kappa}$ from the physical values obtained for $\bar{\kappa} = \kappa_a$ to the corresponding ones obtained for $\bar{\kappa} = \kappa_b$. Actually, ϵ can vary considerably, almost linearly, whereas the variations of G are very small [15]. One has, in particular, $G_a \simeq G_b$, as can be appreciated in fig. 1 for the two states of the neutron SD of the ^{40}Ca nucleus. Thus, the effect of the $G-\kappa$ term in eq. (11) on the function G seems to be small. In fact, according to our results, these effects can

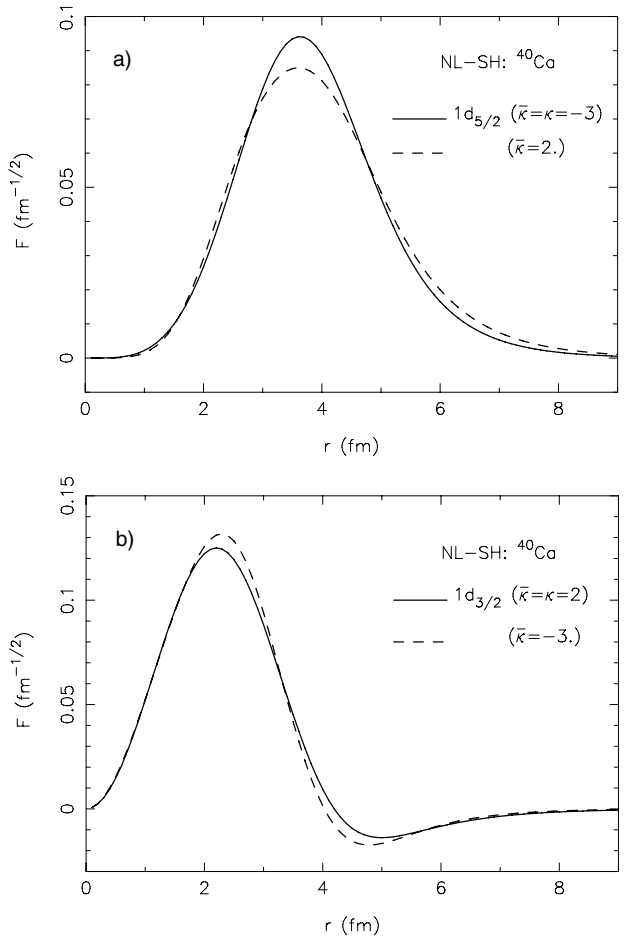

Fig. 2. (a) The wave function F (F_a) for the neutron $1d_{5/2}$ state ($\kappa = -3$) of the ^{40}Ca nucleus, solution of eq. (14) with the values of $\bar\kappa$ indicated in the figure, for the NL-SH set. The case $\bar\kappa = \kappa = -3$ represents the physical state. (b) The same as (a) but now for the wave function F (F_b) corresponding to the neutron $1d_{3/2}$ state ($\kappa = 2$). The case $\bar\kappa = \kappa = 2$ represents the physical state.

be considered almost perturbative, though the spin-orbit splitting for the proton and neutron $1d$ doublets amounts to about 6–7 MeV [19].

To better appreciate the role of the $G - \kappa$ term in eq. (11), it is instructive to analyse not only its effect on the G functions of the SDs, but also its indirect effect on the corresponding F components of the Dirac spinors through eq. (15). This procedure can help us to show up more clearly some symmetry breaking effects. The evolution of the small components F_a and F_b obtained from eq. (14) (or eqs. (11) and (15)) for a SD as $\bar\kappa$ varies from κ_a to κ_b can be observed in figs. 2a and b, respectively. It is seen that the two functions do not change very much with $\bar\kappa$: F_a (for $\bar\kappa = \kappa_b$) $\simeq F_a$ (for $\bar\kappa = \kappa = \kappa_a$) and F_b (for $\bar\kappa = \kappa_a$) $\simeq F_b$ (for $\bar\kappa = \kappa = \kappa_b$). Only in the nuclear surface the differences are more significant, in accordance with what happens also with the large components G_a and G_b in fig. 1. These differences are due to the effect of the $G - \kappa$ term that breaks the spin symmetry (SS). In absence of this term, eq. (11) would not depend on $\bar\kappa$ and, consequently, for the states of a SD, G would be also independent of the value of $\bar\kappa$ (one would have, in particular[1], $G_a \propto G_b$). Then, eq. (15) indicates also that $F_a(\bar\kappa)$ and $F_b(\bar\kappa)$ would not vary with $\bar\kappa$ (one would have, in particular, F_a (for $\bar\kappa = \kappa_b$) $= F_a$ (for $\bar\kappa = \kappa = \kappa_a$) and F_b (for $\bar\kappa = \kappa_a$) $= F_b$ (for $\bar\kappa = \kappa = \kappa_b$)). The results found for SDs seem to be quite natural but, as we shall see in the case of PSDs, they crucially depend on the characteristics of the corresponding symmetry-breaking term.

Choice 2. Equation (11) for the G component can also be obtained from the following Dirac equation:

$$\frac{\mathrm{d}}{\mathrm{d}r}G(r) = -\frac{\bar\kappa}{r}\,G(r) + W\,F(r),$$
$$\frac{\mathrm{d}}{\mathrm{d}r}F(r) = (V + \Delta V')\,G(r) + \frac{\bar\kappa}{r}\,F(r), \tag{16}$$

where

$$\Delta V' = \frac{\kappa(\kappa+1) - \bar\kappa(\bar\kappa+1)}{Wr^2}. \tag{17}$$

Now, from eq. (16), we get

$$F = \frac{1}{W}\left(G' + \frac{\bar\kappa}{r}G\right). \tag{18}$$

Notice that, in eq. (16), it appears $\bar\kappa$ multiplying $G(r)$ and $F(r)$ (not κ) and that for two spin partners and a given value of $\bar\kappa$ (real), eqs. (11), (14) and (16) have identical solutions for ϵ and G. However, the situation is different for F. Whereas for a given value of $\bar\kappa$, eq. (14) has two types of solutions for F, depending on the value of κ chosen, κ_a or κ_b (see eq. (15)), eq. (16) has only one type of solutions (see eq. (18)), though the detailed form depends on $\bar\kappa$. Thus, the number of nodes ($\tilde n_r$) of F in eq. (16) (or in eq. (18)) depends, in principle, on the value of $\bar\kappa$. Figure 3 shows the component F for the $1d$ neutron SD of the ^{40}Ca nucleus obtained from eq. (16) for several values of $\bar\kappa$. We can see that F changes continuously between the two physical functions F_a and F_b, and that the number of nodes ($\tilde n_r$) of F is equal to the number of nodes ($\tilde n_{rb}$) of F_b for values of $\bar\kappa$ such that $\kappa_a < \bar\kappa \leq \kappa_b$.

Our analysis of the behaviour of the solutions of eqs. (11), (14) and (16) for the spin doublets shows that the effect of the SS-breaking $G - \kappa$ term is quite small in all aspects studied. Even if the spin-orbit splittings are large, the $G - \kappa$ term behaves almost as a perturbative term. In this sense, it can be considered, appropriately, as a small symmetry breaking term. In the rest of this section, we shall try to do a similar analysis for the PSS breaking $F - \kappa$ term.

[1] Notice that one would get $G(\bar\kappa) = G_a$ for $\kappa = \kappa_a$ and $G(\bar\kappa) = G_b$ for $\kappa = \kappa_b$ (independently of the value of $\bar\kappa$). Then, as l in eq. (11) takes the same value for $\kappa = \kappa_a$ and $\kappa = \kappa_b$, necessarily, $G_a \propto G_b$.

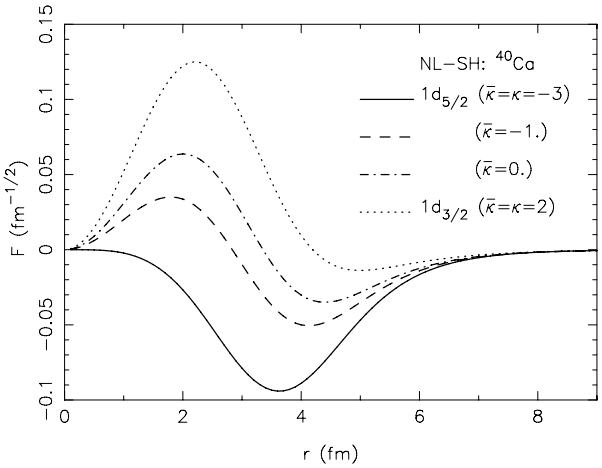

Fig. 3. The wave function F for the neutron SD of the ^{40}Ca nucleus, solution of eq. (16) with the values of $\bar{\kappa}$ indicated in the figure, for the NL-SH set. The cases $\bar{\kappa} = \kappa = -3$ and $\bar{\kappa} = \kappa = 2$ represent the physical states.

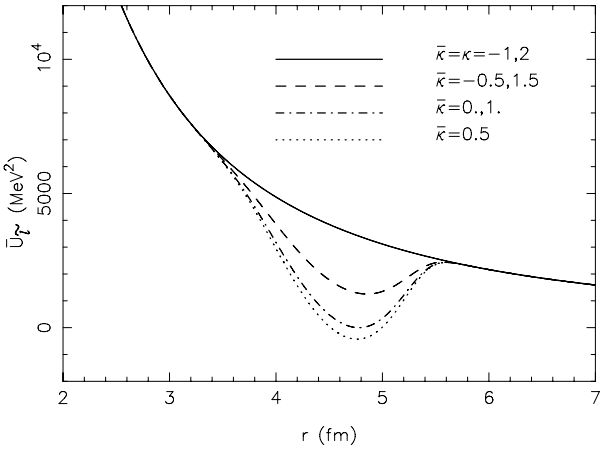

Fig. 4. The modified PCB $\bar{U}_{\tilde{l}}$ given by eqs. (20) and (21) for the indicated values of $\bar{\kappa}$ and the NL-SH parameter set. In eq. (21), the quantity $(r - r_0)$ has been approximated by V/V' (as explained in the text) and we have taken $m = 2$ and $c = 2$ fm^{-2}. The cases $\bar{\kappa} = \kappa = -1, 2$ correspond to the physical PCB.

3.2 Dirac equations equivalent to eq. (12) for F and the pseudospin symmetry

In the same way we have considered two Dirac equations equivalent to eq. (11) for the large component G of the Dirac spinor of a nucleon, we can construct also two Dirac equations formally equivalent to eq. (12) for the corresponding small component F of the Dirac spinor. However, before doing that, we are going to analyse carefully this equation.

Solutions of eq. (12) for F

At first sight, eqs. (11) and (12) look very similar to each other, however, there is an important difference related to the distinct properties of the V and W potentials. Whereas W is a large quantity everywhere in the nucleus, V becomes zero at some point r_0 in the nuclear surface, making the factor V'/V to diverge at r_0. To see the consequence of this fact, we have analysed in detail the behaviour of the solutions of eq. (12) for F in appendix A. As shown there (see also refs. [14,15]), to ensure finite F'' at the singularity point r_0, it is necessary the factor $(F'/F - \bar{\kappa}/r)$ to vanish at this point. Furthermore, to preserve the analyticity of F, it is necessary for $\tilde{l}$ and $\bar{\kappa}$ to be related to each other according to eq. (9). This means, in accordance with eq. (4), that the only acceptable values of $\bar{\kappa}$ for the two states of a PSD are the integer values κ_a and κ_b [2]. Thus, in contrast to the results of eq. (11) found for the large component G of two states of a SD, eq. (12) for the small component F does not allow us to connect two states of a PSD by a continuous variation of $\bar{\kappa}$.

The continuity of the solutions of eq. (12) as $\bar{\kappa}$ varies can be reached by the replacement of κ by $\bar{\kappa}$ in the PCB too, so that the corresponding $\tilde{l}$ and $\bar{\kappa}$ are related to each other in the same way as $\tilde{l}$ and κ are in eq. (9). The problem is that, in this case, the PCB, which is large for small values of r, differs too much from its physical values except when the values of $\bar{\kappa}$ and κ are close to each other. We can solve this problem by modifying the PCB, essentially, around r_0 as, for example, is made in the following equation:

$$-F'' + \left[\frac{V'}{V}\left(\frac{F'}{F} - \frac{\bar{\kappa}}{r}\right) + \frac{\tilde{l}(\tilde{l}+1)}{r^2} + \frac{\bar{\kappa}(\bar{\kappa}-1) - \tilde{l}(\tilde{l}+1)}{r^2}\, e^{-c(r-r_0)^m} + VW\right]F = 0. \quad (19)$$

This equation is formally similar to eq. (12) but with the modified pseudocentrifugal barrier

$$\bar{U}_{\tilde{l}} = U_{\tilde{l}} + \Delta U_{\tilde{l}\bar{\kappa}}, \quad (20)$$

where

$$\Delta U_{\tilde{l}\bar{\kappa}} = \frac{\bar{\kappa}(\bar{\kappa}-1) - \tilde{l}(\tilde{l}+1)}{r^2}\, e^{-c(r-r_0)^m}. \quad (21)$$

Notice that, for the states of a PSD, $\Delta U_{\tilde{l}\bar{\kappa}} = 0$ for $\bar{\kappa} = \kappa_a$ or $\bar{\kappa} = \kappa_b$, but $\Delta U_{\tilde{l}\bar{\kappa}} \neq 0$ for other values of $\bar{\kappa}$. Thus, we can say that this term breaks also the PSS for non-physical situations where $\bar{\kappa} \neq \kappa$ and, consequently, could be also included in a modified or extended PSOP rather than in the modified or extended PCB.

Figure 4 shows the modified PCB $\bar{U}_{\tilde{l}}$ given by eqs. (20) and (21), with $(r - r_0)$ replaced by V/V' (corresponding to the NL-SH parameter set [20][3]), $m = 2$ and $c = 2$ fm^{-2}, for several values of $\bar{\kappa}$. $\bar{U}_{\tilde{l}}$ is appreciably different from $U_{\tilde{l}}$ only near the singularity point r_0. For r far from r_0,

[2] This means, in particular, that the case $\bar{\kappa} = 0$, which would correspond to a case of exact symmetry, cannot be reached with analytical G and F functions if the potential V vanishes inside the nucleus, which always happens for bound states in any realistic model of nucleus [14,15].

[3] Near r_0, $V'/V \simeq r - r_0$ and only depends appreciably on the chosen potential V through r_0.

$\bar{U}_{\tilde{l}} \simeq U_{\tilde{l}}$. However, for $r \to r_0$, $\bar{U}_{\tilde{l}} \to \frac{\bar{\kappa}(\bar{\kappa}-1)}{r^2}$. Thus, F'', obtained from eq. (19), can remain finite and F analytical at the singularity point r_0 for real values of $\bar{\kappa}$.

After the analysis of the solutions of eq. (12) for the function F, which brought us to propose a modification of this equation for the non-physical region of $\bar{\kappa}$, we are going to investigate whether it is possible to find two corresponding equivalent Dirac equations, as we did for the equation for the big component G.

Dirac equations

Choice 1. Equation (19) can be obtained from the Dirac equation (3) by replacing W by $\bar{W}$, defined by the equations:

$$\bar{W}(r) = W(r) + \Delta W(r)\,,$$
$$\Delta W(r) = \frac{V'}{V^2}\frac{\kappa-\bar{\kappa}}{r} + \frac{\bar{\kappa}(\bar{\kappa}-1)-\tilde{l}(\tilde{l}+1)}{Vr^2}\, e^{-c(r-r_0)^m}. \quad (22)$$

The Dirac equation including ΔW reads

$$\frac{\mathrm{d}}{\mathrm{d}r}G(r) = -\frac{\kappa}{r}\,G(r) + (W + \Delta W)\,F(r),$$
$$\frac{\mathrm{d}}{\mathrm{d}r}F(r) = V\,G(r) + \frac{\kappa}{r}\,F(r). \quad (23)$$

Equation (23) allows to obtain the G component from the function F as

$$G = \frac{1}{V}\left(F' - \frac{\kappa}{r}F\right). \quad (24)$$

For $\bar{\kappa} = \kappa$ (integer), eq. (23) and eqs. (19) and (24) are equivalent to the Dirac equation (3). For real values of $\bar{\kappa}$, eqs. (19) and (24) are still formally equivalent to the Dirac equation (23). However, as $V(r_0) = 0$, it is clear that even if F, obtained from eq. (19), is analytical at r_0, G, obtained from eq. (24), becomes divergent at r_0, since, although F satisfies the relation $(F' - \frac{\bar{\kappa}}{r}F)_{r_0} = 0$, the factor $(F' - \frac{\kappa}{r}F)_{r_0} \neq 0$. Thus, in relation with eq. (23), the conditions of analyticity at r_0 for G and F are incompatible if $\bar{\kappa} \neq \kappa$. The factor V^{-1} in eq. (24) behaves as an *amplification factor* of the effects of the PSS breaking on the functions F when these effects are "transmitted" to the corresponding functions G. In this way, we can say that the functions G are more sensible to the PSS breaking effects than the functions F.

The results obtained for PSDs contrast, frontally, with the corresponding results obtained for SDs, where $G_{a,b}(\bar{\kappa})$ and $F_{a,b}(\bar{\kappa})$ change continuously as a function of $\bar{\kappa}$. The reason for these crucial differences is that, whereas the SOP is a smooth quantity, the PSOP is singular in the nuclear surface. Actually, as in the denominator of the SOP there appears the addend $2M$ (through W), the SOP can be considered as a relatively small quantity in relation to the PSOP, which is divergent at r_0. Although the spin-orbit splittings (ΔE_{SO}) in the SP spectra are usually larger than the pseudospin splittings (ΔE_{PSO}), the differences between F_a and F_b for the PSDs are much larger than those between G_a and G_b for the SDs, mainly in the nuclear surface near the singularity point of the PSOP

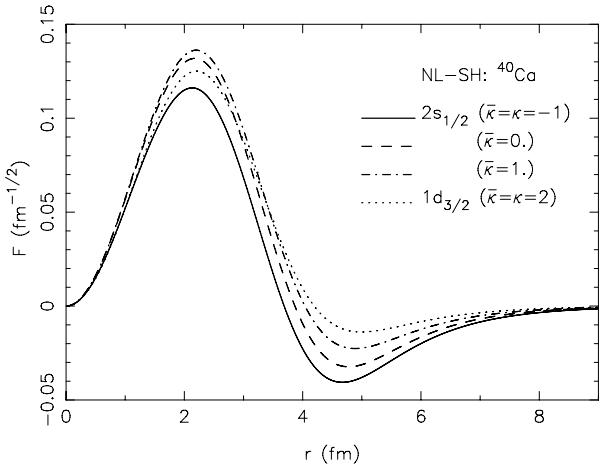

Fig. 5. The wave function F for the neutron PSD of the ^{40}Ca nucleus, solution of eq. (25), or eqs. (19) and (27), for the values of $\bar{\kappa}$ indicated in the figure, and the NL-SH set. For the quantities $r - r_0$, m and c we have used the same criterion as in fig. 4. The cases $\bar{\kappa} = \kappa = -1$ and $\bar{\kappa} = \kappa = 2$ represent the physical states.

(compare figs. 1 and 5[4]). One of the reasons for the inequality $\Delta E_{\mathrm{PSO}} \ll \Delta E_{\mathrm{SO}}$ can be attributed to the fact that the PSOP is, roughly, an odd operator around the singularity point r_0, whereas the SOP is almost an even operator with respect to its point of maximum in the nuclear surface. Actually, if the PSOP in eq. (12) were replaced by its absolute value, its perturbative contribution to the SP energy would diverge and the nucleus would be unbound.

Now, we are going to investigate a second possibility of finding a continuous path, defined also as a parametric function of $\bar{\kappa}$, that allows us to connect the two physical states of a PSD, as we have already done for the SDs. This will help us to better understand the mathematical structure of the Dirac equation in relation with the PSDs.

Choice 2. Equation (12) for the F component can also be obtained from the following Dirac equation:

$$\frac{\mathrm{d}}{\mathrm{d}r}G(r) = -\frac{\bar{\kappa}}{r}\,G(r) + (W + \Delta W')\,F(r),$$
$$\frac{\mathrm{d}}{\mathrm{d}r}F(r) = V\,G(r) + \frac{\bar{\kappa}}{r}\,F(r), \quad (25)$$

where

$$\Delta W'(r) = \frac{\kappa(\kappa-1) - \bar{\kappa}(\bar{\kappa}-1)}{Vr^2}[1 - e^{-c(r-r_0)^m}]. \quad (26)$$

Now, from eq. (25), we get

$$G = \frac{1}{V}\left(F' - \frac{\bar{\kappa}}{r}F\right). \quad (27)$$

[4] Notice that the functions F in fig. 5 have the number of nodes $\tilde{n}_r = 2$, whereas the function G in fig. 1 has the number of nodes $n_r = 1$, and that the difference between the functions of a SD or a PSD decreases as n_r or $\tilde{n}_r$ increases, respectively [15].

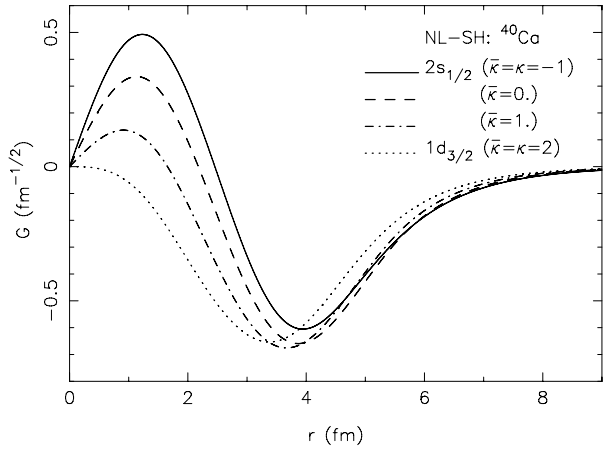

Fig. 6. The big component $G(r)$, corresponding to the cases considered in fig. 5 for the component F.

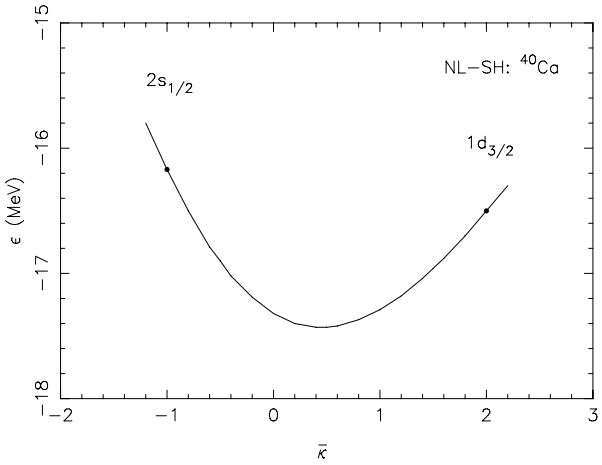

Fig. 7. The single-particle energy ϵ, solution of eq. (25) or eqs. (19) and (27), as a function of $\bar{\kappa}$, for the neutron PSD of the ^{40}Ca nucleus with the NL-SH set. For the quantities $r-r_0$, m and c we have used the same criterion as in fig. 4.

Notice that, as previously in eq. (16), in eq. (25) $\bar{\kappa}$ (rather than κ) appears as a factor multiplying $G(r)$ and $F(r)$, and that for two pseudospin partners and a given value of $\bar{\kappa}$ (real), eqs. (19), (23) and (25) admit, in principle, identical solutions for ϵ and F. However, the two possible functions G obtained from eq. (24) for $\kappa=\kappa_a$ and $\kappa=\kappa_b$ would be divergent at r_0 and, consequently, would not be adequate to describe a physical system. As eq. (16) for F, eq. (25) has only one type of solutions for G (see eq. (27)) and they are analytical. The number of nodes (n_r) of G in eq. (25) depends, in principle, on the value of $\bar{\kappa}$, as shown by eq. (27). As we shall see, n_r remains equal to the number of nodes of G_a (n_{ra}) for values of $\bar{\kappa}$ such that $\kappa_a \leq \bar{\kappa} < \kappa_b$.

The procedure used in this subsection allows us to find solutions of eq. (25), or eqs. (19) and (27), with good asymptotic behaviour for real values of $\bar{\kappa}$. As an example, we present some results for the neutron PSD ($\tilde{l}=1$) of the ^{40}Ca nucleus with the relativistic parameter set NL-SH [20] (in the calculations, the quantity $(r-r_0)$ appearing in eqs. (19) and (26) has been replaced by (V/V'), in accordance with the arguments given above). Figure 5 shows the small component $F(r)$ for several real values of $\bar{\kappa}$. One can see that, for $\kappa_a \leq \bar{\kappa} \leq \kappa_b$, $F(r)$ changes continuously between the physical functions F_a and F_b, remaining appreciably similar to them. Figure 6 shows the corresponding big component $G(r)$ (for the same real values of $\bar{\kappa}$ as in fig. 5). One can see that as $\bar{\kappa}$ increases from κ_a to κ_b, $G(r)$ evolves continuously from G_a to G_b, the number of nodes n_r remaining equal to the number of nodes n_{ra} of $G_a(r)$ for $\kappa_a \leq \bar{\kappa} < \kappa_b$. Thus, eq. (25) (or eqs. (19) and (27)) allows us to connect, through a continuous way, the two physical states of a PSD, in opposition to eq. (23) (or eqs. (19) and (24)) that does not allow this connection. Figure 7 shows the SP energy ϵ obtained from eq. (25) (or eq. (19)) as a function of $\bar{\kappa}$. It is remarkable that ϵ is a continuous function of $\bar{\kappa}$ that crosses the SP energies ϵ_a and ϵ_b corresponding to the physical values. We remind also that this does not happen with the eigenvalues of eq. (23) (or eqs. (12) or (19) with (24), if the analyticity of G is required), they do not cross the physical values as we have explained above (see also refs. [14,15]).

4 The PSS breaking term and the PSS

Equation (19) for the function F allows us also to identify a modified PSS breaking term ($\overline{\text{PSOP}}$) that produces, in combination with the Dirac equation (25), moderate changes in the energies and in the components F of the PSDs as $\bar{\kappa}$ varies continuously from $\bar{\kappa}=\kappa_a$ to $\bar{\kappa}=\kappa_b$. Although it does not allow to connect the physical states a and b in a perturbative way (according to our results of ref. [13]), it allows a smooth transition between the states a and b. In accordance with eq. (19), the $\overline{\text{PSOP}}$ can be defined as

$$\bar{U}_{\bar{\kappa}} = U_{\bar{\kappa}} + \Delta U_{\tilde{l}\bar{\kappa}}, \tag{28}$$

where we remind that $U_{\bar{\kappa}} = \frac{V'}{V}\frac{\bar{\kappa}}{r}$ is the PSOP (with $\kappa \to \bar{\kappa}$) and $\Delta U_{\tilde{l}\bar{\kappa}}$ is defined by eq. (21). Notice that the term $\Delta U_{\tilde{l}\bar{\kappa}}$ near $r=r_0$, where it is relevant, behaves as the PCB $U_{\tilde{l}}$ (since the PSOP is divergent at r_0, the addend $\Delta U_{\tilde{l}\bar{\kappa}}$ represents only a very small fraction of the PSOP around r_0). Thus, we have got a $\overline{\text{PSOP}}$, $\bar{U}_{\bar{\kappa}}$, that, in combination with eq. (25), is weaker than the PSOP $U_{\bar{\kappa}}$. Our results in appendix A and in the previous section indicate that the PSS breaking term and the PCB must be adequately related to each other to guarantee analytical properties for the components F and G of the Dirac spinors around r_0, although, for real values of $\bar{\kappa}$, it is only possible in combination with the Dirac equation (25). In relation with the Dirac equation (23), the $\overline{\text{PSOP}}$ is still, essentially, as strong as the PSOP. In accordance with our results, we conclude that, at any circumstances, the magnitude of the PSOP cannot be considered much smaller than the PCB, in opposition to what is claimed erroneously in refs. [6–9]. On the contrary, since they must be related to each other in accordance with eq. (9) to guarantee an acceptable behaviour of the G and F components of the Dirac spinor

in the nuclear surface, the PSOP and the PCB must be considered at a similar level of relevance in eq. (12).

As we have discussed in ref. [15], the strength of the PSOP makes it difficult to explain the approximate PSS observed in nuclei starting from a situation of exact PSS as that considered in refs. [3–5]: $\Sigma_S+\Sigma_0=0$, or in ref. [15] for $U_{\bar{\kappa}}$ with $\bar{\kappa}<\kappa_a$, by our group, since the remaining way until the physical situation with realistic values of $\Sigma_S+\Sigma_0$ and $U_{\bar{\kappa}}$ cannot be realized in a perturbative way in any of the two cases.

We have also concluded in refs. [13–18], using different arguments, that the similarity found in the relativistic calculations between F_a and F_b for the two states of a PSD, cannot justify that $\epsilon_a \simeq \epsilon_b$ (which contradicts the conclusions of refs. [3–9]). One of the arguments that support this statement can be found in the results of sect. 2 that indicate that G_a and G_b for the partner states of the SDs are more similar than F_a and F_b for the partner states of the PSDs[5], in spite of the fact that the spin-orbit splittings for SDs are generally larger than the pseudospin-orbit splittings for PSDs. We should not forget also that the condition $\epsilon_a=\epsilon_b$ for the two states of a PSD is compatible with $F_a \neq F_b$. In fact, the equality $\epsilon_a=\epsilon_b$ implies, necessarily, that $F_a \neq F_b$ (actually, F_a and F_b cannot be even proportional to each other) [13,15]. We have seen in sect. 2, that the progressive quenching of the SOP produces small and continuous modifications of the eigenenergies and eigenfunctions, so that ϵ_a approaches to ϵ_b and G_a becomes proportional to G_b for SDs. However, the quenching of the PSOP produces discontinuities both in the energies and in the wave functions.

[5] In fact, to be more complete, one should compare also the differences between G_a and G_b for a SD with the effects of the differences between F_a and F_b for a PSD on the corresponding G_a and G_b functions obtained from eq. (24) (which is compatible with eq. (3)). For example, in the case of exact PSS (if it were possible) we should have $\epsilon_a=\epsilon_b$ and $F_a \propto F_b$ and, for $\kappa=\kappa_a$, we would obtain $F_a \to G_a=\frac{1}{V}\left(F_a'-\frac{\kappa_a}{r}F_a\right)$, $F_b \to G=\frac{1}{V}\left(F_b'-\frac{\kappa_a}{r}F_b\right) \propto G_a+\Delta G_a$, with $\Delta G_a=0$. In the same way, for $\kappa=\kappa_b$, we would obtain $F_b \to G_b$, $F_a \to G \propto G_b+\Delta G_b$, with $\Delta G_b=0$. But, since for realistic models of nuclei $V(r_0)=0$, for some point r_0 in the nuclear surface, and $\kappa_a \neq \kappa_b \Rightarrow |\Delta G_a| \to \infty$ or/and $|\Delta G_b| \to \infty$ for $r \to r_0$. This result indicates that the exact PSS ($\epsilon_a=\epsilon_b$, $F_a \propto F_b$) is incompatible with nuclear models in which $V(r_0)=r_0$, *i.e.*, with models giving bound PSDs. In real nuclei, with approximate PSS, we have for $r \to r_0$: $|\Delta G_a(r)| \to \infty$ and $|\Delta G_b(r)| \to \infty$. Notice that this is a particular case of the result given in the second part of sect. 2 (choice 1). A similar analysis could be made by comparing the differences between F_a and F_b for a PSD with ΔF_a and ΔF_b obtained from eq. (18) with the functions G of a SD in the same way as we obtained ΔG_a and ΔG_b from the functions F of a PSD. The result can be observed by comparing fig. 5 with figs. 2a and b. We can see that in this case $|\Delta F_a|$, $|\Delta F_b| \ll |F_a-F_b|$. Thus, it is quite clear that the effect of the PSS breaking term is much larger than the effect of the SS breaking term, in contradiction to what was generally believed until we questioned that in refs. [14,15].

5 New strategy to explain the PSS

The facts just discussed in the previous paragraph indicate that *the conditions* $\epsilon_a \simeq \epsilon_b$ and $F_a \simeq F_b$, *for the two states of a* PSD, *are not strictly dependent on each other*. We can use this fact as an argument to recover the original concept of PSS as a degeneracy of the PSDs, no matter whether F_a and F_b are proportional to each other or not. That is why we have proposed in ref. [16] an explanation of the PSS starting from a situation of exact degeneracy of the PSDs but where F_a and F_b are not proportional to each other. The starting model satisfies $\Sigma_S-\Sigma_0=0$ (*i.e.*, there is no spin-orbit interaction), whereas $\Sigma_S+\Sigma_0$ is a harmonic-oscillator potential. This model, defined by the Hamiltonian H_0, produces degenerate SDs with $G_a \propto G_b$ and degenerate PSDs but with F_a non-proportional to F_b [10]. Then, although a more realistic quantity $\Sigma_S+\Sigma_0$ breaks the degeneracy of the PSDs, the choice of appropriate values for $\Sigma_S-\Sigma_0 \neq 0$ partially restores the quasi-degeneracy of the PSDs. Thus, we can say that there is a continuous (*almost perturbative*[6]) way that connects the model satisfying exact degeneracy of the PSDs with realistic models of nuclei with approximate PSS.

Since the consideration of the PSOP as the PSS breaking term requires the two conditions $\epsilon_a \simeq \epsilon_b$ and $F_a \propto F_b$ to be directly dependent on each other, our present form of understanding the PSS indicates that the PSOP is not the appropriate choice for the PSS breaking term in this model. Let us see how we can define an appropriate new one in accordance with the present interpretation of the PSS. Let us write the SP Hamiltonian corresponding to eq. (1) as $H=H_0+H_1$, where H_0 exhibits exact degeneracy of PSDs and H_1 represents the corresponding breaking term.

If the Dirac equation corresponding to H is given by eq. (3), and we write $W=W_0+W_1$, $V=V_0+V_1$ and $\epsilon=\epsilon_0+\epsilon_1$, where W_0, V_0 and ϵ_0 represent, respectively, the potentials entering eq. (3) and the SP binding energy (with opposite sign) corresponding to H_0, we have $W_0=2M+\epsilon_0$ and $V_0=\Omega-\epsilon_0$, Ω being an appropriate harmonic-oscillator potential for a given nucleus, and $W_1=\Sigma_S-\Sigma_0+\epsilon_1$ and $V_1=\Sigma_S+\Sigma_0-\Omega-\epsilon_1$.

From the Dirac equation, we get the following equivalent equation for the small component F:

$$-F''+\left[\frac{V_0'+V_1'}{V_0+V_1}\left(\frac{F'}{F}-\frac{\kappa}{r}\right)+\frac{\tilde{l}(\tilde{l}+1)}{r^2} + (V_0+V_1)(W_0+W_1)\right]F=0\,, \tag{29}$$

where the $F-\kappa$ term breaks explicitly the relation $F_a \propto F_b$, even if $V_1=W_1=0$. However, in this case, we have $\epsilon_{a0}=\epsilon_{b0}$ [10] (indicating that the $F-\kappa$ term is not the appropriate PSS breaking term). The fact

[6] This connection is made in two steps: the change of the harmonic-oscillator potential to a more realistic one and the inclusion of the SOP. Both steps can be considered almost perturbative (we have realized above that this is the case for the SOP).

that the $F-\kappa$ term is not a perturbative term facilitates this equality[7]. The self-consistency effects due to the $F-\kappa$ term, which breaks the proportionality between F_a and F_b, exactly compensate the contribution of the $F-\kappa$ term to the pseudospin-orbit splittings. It is important to remind that this situation of exact degeneracy of the PSDs is compatible with bound nuclei, on the contrary to what happens with the exact PSS considered in refs. [3–5]. When the potentials V_1 and W_1 entering eq. (29) are considered, their effects are almost perturbative and the real nuclei are recovered.

The model can explain the relation $\epsilon_a \simeq \epsilon_b$ for the pseudospin partners, but, how can we explain the relation $F_a \simeq F_b$ found also in relativistic calculations for PSDs with $\tilde{n}_r \geq 3$? The similarity between F_a and F_b for the two states of a PSD in the inner region of the nuclei can be explained, independently of $\epsilon_a \simeq \epsilon_b$, because the effects of the $F-\kappa$ term, responsible for the lack of proportionality between F_a and F_b, are mainly important in the nuclear surface due to the singularity of the $F-\kappa$ term at r_0. Furthermore, since the role of the terms entering F' and, mainly, F'' in eq. (29) increases with the number of nodes ($\tilde{n}_r$) of F [15], F_a and F_b become more similar as $\tilde{n}_r$ increases and, generally, the same happens with ϵ_a and ϵ_b (although the relation $F_a \simeq F_b$ cannot justify the small splitting of PSDs found in calculations [13,15]). This can be understood taking into account that the contribution of the $F-\kappa$ term in eq. (29) to the $\Delta E_{\rm PSO}$ is smaller and, quite generally, with a sign opposite to that of the rest of the terms entering this equation [13,15], which only depends on the differences between F_a and F_b and ϵ_a and ϵ_b (to a minor degree, through V).

It is important to notice that the basis of this explanation for the PSS is not the relativistic character of the model used. It can be formulated in similar terms for the non-relativistic models. In this case, the starting point is the non-relativistic SP harmonic-oscillator potential without spin-orbit interaction. This model produces also degenerate SDs and PSDs. When a more realistic SP potential is used, the degeneracy of the PSDs is broken, but it is again partially restored if an adequate spin-orbit interaction is added by hand. Thus, the PSS can be explained in a simple and similar way in the relativistic and non-relativistic approximations, although the spin-orbit interaction, which plays an important role in this explanation, is more naturally incorporated in the relativistic models. It is important that the explanation of the PSS be valid also in the non-relativistic case, since the quantities V_1 and W_1 cannot be neglected in realistic models in the non-relativistic limit[8]. A more detailed analysis of the considerations presented in this section in relation to the PSS is made in appendix B.

6 On the generality of the results obtained

Although we have presented, as an example, only numerical results for the $(1d_{3/2}, 1d_{1/2})$ neutron SD and the $(2s_{1/2}, 1d_{3/2})$ neutron PSD of the ^{40}Ca nucleus, the qualitative aspects of our conclusions are completely general and can be applied, without restriction, to the neutron or proton SDs and PSDs of any other nucleus (and their validity is not restricted to the Dirac-Hartree approximation we have used here). However, the size of the nucleus does specifically affect the spin and pseudospin doublets. For instance, in the Hartree approximation, the SOP presents only a very weak dependence on the SP energy and, consequently, its form is almost identical for the states of all SDs. As the SOP becomes large in the nuclear surface, the SDs corresponding to the inner nucleons are relatively more affected by the SOP in light nuclei than in heavy ones. Thus, the spin-orbit splitting of a given SD is larger in light nuclei than in heavy ones (although the proton fraction Z/A can have also a non-negligible influence, mainly through the pion contribution in the relativistic Hartree-Fock approximation [19]).

The PSOP, on the contrary to the SOP, strongly depends on the SP energy ϵ. Thus, the singularity point r_0 is determined by the value of ϵ. In particular, as it increases, the value of r_0 increases. Then, each state of a PSD has its corresponding associate value of r_0. Furthermore, around the singularity point, the PSOP behaves always as $(r-r_0)^{-1}$, being, roughly, an odd operator. We have already discussed above some consequences of this behaviour of the PSOP on the splitting of the PSDs. To understand the subtle consequences of all this on the different nuclei is quite difficult. However, there is an important point that distinguishes heavy nuclei from light ones in relation with the spin and pseudospin symmetries. In heavy nuclei, there are SDs and PSDs with larger values of n_r and $\tilde{n}_r$, respectively, than in light nuclei. Thus, concerning the SDs, the role of the terms containing G' and, mainly, of those containing G'' in eq. (6) increases, relatively to the role of the SOP, with the number of nodes (n_r) of G and, consequently, the two functions of a SD, G_a and G_b, become more similar as n_r increases. As we have explained in the previous section, something very similar happens for the two functions F_a and F_b of a PSD as their number of nodes $\tilde{n}_r$ increases. In this case, furthermore, the corresponding energies ϵ_a and ϵ_b approach each other. This fact can be also explained by the relative minor relevance of the $F-\kappa$ term [16]. However, the splittings of the SDs do not decrease, in general, as n_r increases. This can be explained because the states of a SD with large values of n_r take relevant values closer to the nuclear surface (where the SOP is large) than the states of a SD with small values of n_r.

We have seen for the ^{40}Ca nucleus that the G and F functions of the partner states of the $1d$ SD and their corresponding energies vary continuously as functions of

[7] Notice that our actual form of understanding the PSS is based on the fact that the $F-\kappa$ term is large rather than small. The case considered here supplies also an example of the independence between the relations $\epsilon_{a0} \simeq \epsilon_{b0}$ and $F_a \propto F_b$.

[8] Notice also that the PSOP can be considered neither exclusively as a relativistic term in the sense that it cannot be neglected in the equation for the function F as M becomes very large.

κ when κ is allowed to vary continuously as a real number ($\bar{\kappa}$). This feature remains valid for the neutron and proton SDs of all nuclei, independently of the value of n_r. The discontinuity of ϵ and F (as solutions of eq. (12)), found for the neutron PSD of the ^{40}Ca nucleus when $\bar{\kappa}$ varies continuously from κ_a to κ_b, is also a general result valid, in particular, for all PSDs with larger values of $\tilde{n}_r$, though the similarity of the wave functions F_a and F_b of a PSD strongly increases with $\tilde{n}_r$. This is so because the PSOP remains singular at r_0.

7 Conclusions

We have studied, in a comparative way, the effects of the spin and pseudospin symmetry breaking terms SOP and PSOP, respectively, on the energies and the large and small components of the Dirac spinors corresponding to the spin and pseudospin doublets. We have investigated, in particular, the analytical properties of the small component F of the Dirac spinor for PSDs around the singularity point of the PSOP. We have shown that the PSOP and the PCB must be appropriately related to each other to describe adequately the SP wave functions in the nuclear surface. The relation between $\tilde{l}$ and κ, $\tilde{l}(\tilde{l}+1) = \kappa(\kappa-1)$, for the PSDs is necessary (but not always sufficient) to get solutions of the Dirac equation with analytical behaviour for G and F in the nuclear surface where the PSOP is singular. However, the situation is not the same for the centrifugal barrier and the SOP for the SDs. In this case, the relation $l(l+1) = \kappa(\kappa+1)$ does not play an important role. Even if this relation is not fulfilled as, for example, when κ varies as a real parameter in the SOP, its effect is almost perturbative (in particular, the variations of the components G and F of the Dirac spinors for the SDs are very small).

We have determined a modified PSOP that produces smaller effects on the energy and wave function than the PSOP when the parameter $\bar{\kappa}$ appearing in it is left to vary in a domain of real values rather than keeping it equal to its physical (integer) value. This has allowed us to define a continuous way to connect the two states of a PSD as $\bar{\kappa}$ varies continuously between the two physical values of κ, κ_a and κ_b, corresponding to these two states.

The strong effect of the PSOP, as κ is allowed to vary as a real number ($\bar{\kappa}$), contrasts with the fact that the SOP is a smooth function of κ, although the splittings of the SDs are generally larger than the splittings of the PSDs. The strong differences found between the effects of the SOP and PSOP on the energies and wave functions of the partner states of the SDs and PSDs, respectively, indicate that the approximate PSS found in real nuclei is more easily explained starting from a model that requires the degeneracy of PSDs but not the equality (or even not the proportionality) of the small F components of the pseudospin partners. This is possible since the degeneracy of the PSDs does not require the equality (or proportionality) of the small F functions of the two partners, on the contrary, they must be non-proportional to each other.

Thus, the approximations $\epsilon_a \simeq \epsilon_b$ and $F_a \simeq F_b$ are not strictly dependent on each other (as happens for the energies and G functions of the SDs). This allows us to give a simple explanation of the PSS, starting from an oversimplified nuclear model where nucleons move in a harmonic-oscillator potential without spin-orbit interaction, which presents exact degeneracy for PSDs but with $F_a \neq F_b$ (though $F_a \simeq F_b$). The inclusion of a more realistic potential and of the spin-orbit interaction breaks the degeneracy of PSDs, but still keeps $\epsilon_a \simeq \epsilon_b$ and $F_a \simeq F_b$, and these latter approximate relations are improved as $\tilde{n}_r$ increases. It is worth stressing that this explanation is valid both for relativistic and for non-relativistic models. This is important, since the terms breaking the PSS cannot be considered as exclusively relativistic.

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Appendix A. Analytical behaviour of the F function near the singularity point r_0

In this appendix, we explore the analytical properties of the small component F of the Dirac spinor as a solution of eq. (12). We let $\bar{\kappa}$ to take values between κ_a and κ_b corresponding to the two values of κ of the partner states of a PSD. As we will see, F is analytical only if $\bar{\kappa}$ takes the physical values $\kappa_a = -l_a - 1$ and $\kappa_b = l_b$. For other values of $\bar{\kappa}$, there is a logarithmic term which destroys the analyticity. Nevertheless, F and its first derivative F' are continuous everywhere.

A very important fact related to eq. (12) is that the quantity $V(r)$ vanishes at some point r_0 for bound states ($\epsilon < 0$). To see how $F(r)$ behaves near the singularity point r_0 let us expand it in powers of $\delta = r - r_0$:

$$F(r) = \sum_{k=0}^{\infty} F_k \delta^k + \delta^2 \ln \delta \sum_{k=0}^{\infty} f_k \delta^k. \tag{A.1}$$

To prove that this representation is compatible with eq. (12), we also expand the coefficients of eq. (12) into the powers of δ as follows:

$$\begin{aligned}
&\frac{V'}{V} = \frac{1}{\delta} + \sum_{k=0}^{\infty} \alpha_k \delta^k, \\
&\frac{\bar{\kappa}}{r} \cdot \frac{V'}{V} = \frac{\bar{\kappa}}{r_0} \cdot \frac{1}{\delta} + \sum_{k=0}^{\infty} \beta_k \delta^k, \\
&\frac{\tilde{l}(\tilde{l}+1)}{r^2} = \frac{\tilde{l}(\tilde{l}+1)}{r_0^2} + \sum_{k=1}^{\infty} \gamma_k \delta^k, \\
&VW = \sum_{k=1}^{\infty} \eta_k \delta^k.
\end{aligned} \tag{A.2}$$

If we substitute the expansions (A.1) and (A.2) into eq. (12), collect all terms proportional to δ^n and $\delta^n \ln \delta$

and put their respective coefficients equal to zero, we get the following recurrence relations:

$$n(n+2)f_n = \Bigg\{ -\frac{\bar\kappa}{r_0} f_{n-1} + \frac{\tilde l(\tilde l+1)}{r_0^2} f_{n-2} + f_0(2\alpha_{n-1} - \beta_{n-2} + \gamma_{n-2} + \eta_{n-2}) + \sum_{m=1}^{n-1} f_m(3\alpha_{n-m-1} - \beta_{n-m-2} + \gamma_{n-m-2} + \eta_{n-m-2}) \Bigg\}, \tag{A.3}$$

$$n(n-2)F_n = \Bigg\{ -\frac{\bar\kappa}{r_0} F_{n-1} + \frac{\tilde l(\tilde l+1)}{r_0^2} F_{n-2} - F_0(\beta_{n-2} - \gamma_{n-2} - \eta_{n-2}) + \sum_{m=1}^{n-1} F_m(m\alpha_{n-m-1} + \alpha_{n-m-3} - \beta_{n-m-2} + \gamma_{n-m-2} + \eta_{n-m-2}) - 2(n-1)f_{n-2} + \sum_{m=0}^{n-3} \alpha_{n-m-3} f_m \Bigg\}. \tag{A.4}$$

In relations (A.3) and (A.4), the quantities f, F, α, β, γ and η are assumed to be equal to zero if their indices are negative, and besides $\gamma_0 = \eta_0 = 0$, which follows from eq. (A.2). Putting $n = 1$ in eq. (A.4) one obtains

$$F_1 = \frac{\bar\kappa}{r_0} F_0. \tag{A.5}$$

This relation ensures that the second term in eq. (12) is finite at $r = r_0$. Setting $n = 0$ in eq. (A.3) yields

$$f_0 = \frac{\tilde l(\tilde l+1) - \bar\kappa(\bar\kappa - 1)}{2r_0^2} F_0, \tag{A.6}$$

if one takes into account that

$$\alpha_0 = \frac{V''(r_0)}{2V'(r_0)}$$

and

$$\beta_0 = -\frac{\bar\kappa}{r_0^2} + \frac{\bar\kappa}{r_0} \cdot \frac{V''(r_0)}{2V'(r_0)}.$$

It is worth noting that F_2 cannot be obtained from eq. (A.4). Then, F_0 and F_2 are to be considered free. Thus, we have a peculiar situation: we cannot arbitrarily fix the value of the solution of eq. (12) and its first derivative at the point r_0 though it is a second-order equation. Instead, $F(r_0)$ and $F''(r_0)$ can be chosen freely. Then, all other F_n and f_n coefficients are calculated using eq. (A.3) and eq. (A.4). Such property of eq. (12) is related to the singularity of its coefficients at $r = r_0$. From eq. (A.6), we can conclude that if the value of $\bar\kappa$ is not equal to κ_a or κ_b, then, the solution of eq. (12) is not analytical because of the logarithmic term in eq. (A.1). One sees from expression (A.1) that $F(r)$ and its first derivative are continuous at r_0 but the second- and higher-order derivatives are infinite.

Appendix B. Analysis of the behaviour of the solutions of eq. (29) for F

We analyse, in a qualitative way, some important features of the solutions of the eq. (29) for F,

$$-F'' + \left[\frac{V_0' + V_1'}{V_0 + V_1}\left(\frac{F'}{F} - \frac{\kappa}{r}\right) + \frac{\tilde l(\tilde l+1)}{r^2} + (V_0 + V_1)(W_0 + W_1)\right] F = 0, \tag{B.1}$$

in relation with the PSS. The potentials V_0, V_1, W_0 and W_1 satisfy the relations $W = W_0 + W_1$ and $V = V_0 + V_1$. As explained in sect. 5, they are defined so that if we take $W_1 = V_1 = 0$, the remaining part H_0 of the Dirac Hamiltonian given by eq. (1) exhibits exact degeneracy of PSDs (though $F_a \neq F_b$). We have, in particular, $W_0 = 2M + \epsilon_0$ and $V_0 = \Omega - \epsilon_0$, Ω being an appropriate harmonic-oscillator potential for a given nucleus and ϵ_0 the SP binding energy (with opposite sign) corresponding to H_0.

Firstly, we take into account that near the singularity point r_0,

$$\frac{V_0' + V_1'}{V_0 + V_1} \simeq \frac{1}{r - r_0} \tag{B.2}$$

and

$$\frac{V_0'}{V_0} \simeq \frac{1}{r - r_{00}}, \tag{B.3}$$

where r_{00} is defined by the condition $V_0(r_{00}) = 0$. Then, admitting that $r_{00} \simeq r_0$[9] and that the main contribution of the term involving the factor $(V_0' + V_1')/(V_0 + V_1)$ comes from the region near r_0, where it is singular, we can replace this factor by the quantity V_0'/V_0. The resulting equation is

$$-F'' + \left[\frac{V_0'}{V_0}\left(\frac{F'}{F} - \frac{\kappa}{r}\right) + \frac{\tilde l(\tilde l+1)}{r^2} + (V_0 + V_1)(W_0 + W_1)\right] F \simeq 0. \tag{B.4}$$

Now, to study the solutions of this equation, we write $F = F_0 + F_1$, where F_0 and F_1 satisfy the equations

$$-F_0'' + \left[\frac{V_0'}{V_0}\left(\frac{F_0'}{F_0} - \frac{\kappa}{r}\right) + \frac{\tilde l(\tilde l+1)}{r^2} + V_0 W_0\right] F_0 = 0, \tag{B.5}$$

and

$$-F_1'' + \left[\frac{V_0'}{V_0}\left(\frac{F_1'}{F_1} - \frac{\kappa}{r}\right) + \frac{\tilde l(\tilde l+1)}{r^2} + (V_0 + V_1)(W_0 + W_1)\right] \times F_1 + (V_0 W_1 + V_1 W_0 + V_1 W_1) F_0 \simeq 0, \tag{B.6}$$

respectively.

[9] Notice that, in principle, it is possible to choose the harmonic-oscillator potential Ω so that $r_{00} = r_0$.

If the κ term entering this equation is taken from eq. (B.5), we have

$$-F_1'' + \frac{F_0''}{F_0}F_1 + \left[\frac{V_0'}{V_0}\left(\frac{F_1'}{F_1} - \frac{F_0'}{F_0}\right) + V_0W_1 + V_1W_0 + V_1W_1\right]F_1 + (V_0W_1 + V_1W_0 + V_1W_1)F_0 \simeq 0. \quad \text{(B.7)}$$

We know that $\epsilon_{a0} = \epsilon_{b0}$, but as $F_{a0} \neq F_{b0}$ we will have $\epsilon_{a1} \neq \epsilon_{b1}$ and $F_{a1} \neq F_{b1}$. However, since as $\tilde{n}_r$ increases, F_{a0} approaches F_{b0}, it will happen that ϵ_{a1} approaches ϵ_{b1} and F_{a1} approaches F_{b1}. Consequently, ϵ_a will approach ϵ_b and the PSS will be improved as $\tilde{n}_r$ increases, as confirmed by calculations [15]. Thus, the quasi-degeneracy of the PSDs and the similarity of F_a and F_b can be explained in a simple way, starting from an oversimplified nuclear model where nucleons move in a harmonic-oscillator potential without spin-orbit interaction, which presents exact degeneracy for PSDs but with $F_a \neq F_b$ (though $F_a \simeq F_b$). The inclusion of a more realistic potential and of the spin-orbit interaction breaks the degeneracy of PSDs, but still $\epsilon_a \simeq \epsilon_b$ and $F_a \simeq F_b$, and these approximations are improved as $\tilde{n}_r$ increases.

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